Liljencrants-Fant Glottal Wave Model Implementation

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15 June 2016

Abstract

Here we present the numerical method used for generating Liljencrants-Fant glottal flow implemented in Acwato software (available at acwato.com). The computation time for calculating a complete LF glottal cycle does not exceeds $2\,10^{-4}\,sec$ on an Intel core i7 processor running at 4.7 GHz . This makes it almost suitable for real-time articulatory speech synthesis. .

1 Introduction

The Liljencrants-Fant glottal wave model is defined by the derivative $U_g'(t)$ of the glottal signal $U_g(t)$:

$$U_g'(t) = \begin{cases} -Ee^{a(t-T_e)}\frac{\sin(\omega_g t)}{\sin(\omega_g T_e)} & 0 < t < T_e \\ -\frac{E}{\beta T_a}(e^{-\beta(t-T_e)} - e^{-\beta(T_0-T_e)}) & T_e < t < T_0 \end{cases} \tag{1}$$

with

$$\omega_g = \frac{\pi}{T_p} \tag{2}$$

T_p is the peak time where the derivative is 0 and the amplitude is maximum.

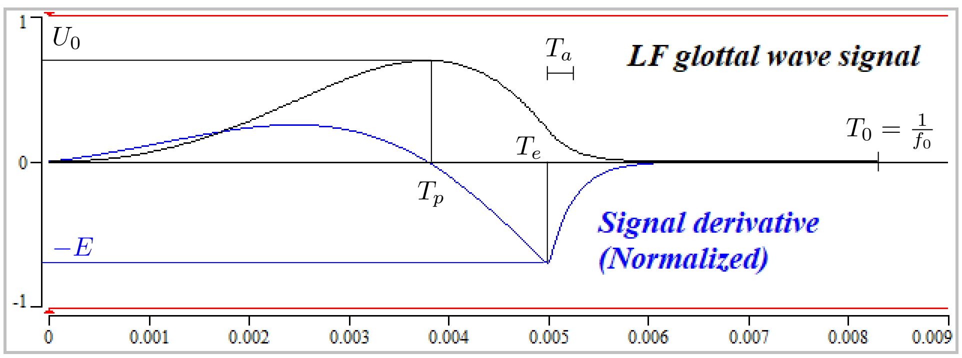

Figure 1: One cycle of the Liljencrants-Fant glottal wave signal. The derivative is normalized for the plot so that the signal and its derivative have the same amplitude.

From now, we indicat by the subscript l the left part of 1 and by r its right part

Since we want continuity between the left part ($t < T_e$) and the right one ($t > T_e$) and therefore, $U_{gr}'(T_e) = -E$, we have

$$\beta T_a = 1 - e^{-\beta(T_0-T_e)} \tag{3}$$

$$T_a - \frac{1 - e^{-\beta(T_0-T_e)}}{\beta} = 0 \tag{4}$$

Once T_a is provided, β is obtained by solving 4 using Newton's method using $\frac{1}{T_a}$ as start value for β.

Since

$$\int g(x)e^{ax}dx = \frac{1}{a}\{g(x)e^{ax} - \int g'(x)e^{ax}dx\}$$

If we pose:

$$E_a = \frac{-Ee^{-aT_e}}{\sin(\omega_g T_e)} \tag{5}$$

$$
\begin{aligned}
U_{gl} &= \int -Ee^{a(t-T_e)}\frac{\sin(\omega_g t)}{\sin(\omega_g T_e)}dt \\
&= E_a \int \sin(\omega_g t)e^{at}dt \qquad (6) \\
&= E_a \frac{1}{a}\{\sin(\omega_g t)e^{at} - \int \omega_g \cos(\omega_g t)e^{at}dt + C_l\} \\
&= E_a \frac{1}{a}\{sin(\omega_g t)e^{at} - \frac{1}{a}\{\omega_g \cos(\omega_g t)e^{at} + \omega_g^2 \int \sin(\omega_g t)e^{at}dt + C_l\}\} \qquad (7)
\end{aligned}
$$

$\Leftrightarrow (6 = 7)$

$$(1+\frac{\omega_g^2}{a^2})E_a \int \sin(\omega_g t)e^{at}dt = E_a \frac{1}{a}\{sin(\omega_g t)e^{at} - \frac{\omega_g}{a}\cos(\omega_g t)e^{at} + C_l\} \tag{8}$$

$\Leftrightarrow$ (substitute 8 in 6)

$$
\begin{aligned}
U_{gl}(t) &= E_a \int \sin(\omega_g t)e^{at}dt \qquad (9) \\
&= \frac{E_a}{1+\frac{\omega_g^2}{a^2}}\frac{1}{a}\{\sin(\omega_g t)e^{at} - \frac{\omega_g}{a}\cos(\omega_g t)e^{at} + C_l\} \\
&= \frac{E_a}{a^2+\omega_g^2}\{a\sin(\omega_g t)e^{at} - \omega_g \cos(\omega_g t)e^{at} + aC_l\} \qquad (10)
\end{aligned}
$$

Since we want $U_{gl}(0) = 0$, we have $aC_l = \omega_g$. posing

$$F_a = \frac{E_a}{a^2+\omega_g^2} \tag{11}$$

$$U_{gl}(t) = F_a(\omega_g + a\sin(\omega_g t)e^{at} - \omega_g \cos(\omega_g t)e^{at}) \tag{12}$$

Now, we integrate the right part

$$
\begin{aligned}
U_{gr}(t) &= \int -\frac{E}{\beta T_a}(e^{-\beta(t-T_e)} - e^{-\beta(T_0-T_e)}) \\
&= -\frac{E}{\beta T_a}(-\frac{1}{\beta}e^{-\beta(t-T_e)} - e^{-\beta(T_0-T_e)}t) + C_r \qquad (13)
\end{aligned}
$$

Now we want $U_{gr}(T_0) = 0$ to ensure continuity beteen two cyles.

$$\Rightarrow C_r = -\frac{E}{\beta T_a}(\frac{1}{\beta} + T_0)e^{-\beta(T_0-T_e)} \tag{14}$$

$$U_{gr}(t) = \frac{E}{\beta T_a}\{\frac{1}{\beta}e^{-\beta(t-T_e)} + e^{-\beta(T_0-T_e)}(t - \frac{1}{\beta} - T_0)\} \tag{15}$$

Since we want continuity in T_e, we have $U_{gl}(T_e) = U_{gr}(T_e)$:

$$F_a\{\omega_g + a\sin(\omega_g T_e)e^{aT_e} - \omega_g \cos(\omega_g T_e)e^{aT_e}\} = \frac{E}{\beta T_a}\{\frac{1}{\beta} + e^{-\beta(T_0-T_e)}(T_e - \frac{1}{\beta} - T_0)\} \tag{16}$$

$\Leftrightarrow$ Using 5 and 11

$$\frac{1}{a^2+\omega_g^2}\{\frac{\omega_g e^{-aT_e}}{\sin(\omega_g T_e)} + a - \omega_g\frac{\cos(\omega_g T_e)}{\sin(\omega_g T_e)}\} = \frac{-1}{\beta T_a}\{\frac{1}{\beta} + e^{-\beta(T_0-T_e)}(T_e - \frac{1}{\beta} - T_0)\} \tag{17}$$

Now, we want to fix the signal amplitude U_0 where U_{gl} is maximum when $t = T_p$ and therefore:

$$\sin(\omega_g T_p) = 0 \Rightarrow T_p = \frac{\pi}{\omega_g} \tag{18}$$

$$U_0 = U_{gl}(T_p) = F_a\omega_g(1 + e^{aT_p}) = F_a\omega_g(1 + e^{a\frac{\pi}{\omega_g}}) \tag{19}$$

Therefore, E and a are obtained by solving the non linear system of equations provided by equations 17 and 19 in which F_a has bee replaced by its definition (11 and 5).

$$\begin{cases} \frac{1}{a^2+\omega_g^2}\{\frac{\omega_g e^{-aT_e}}{\sin(\omega_g T_e)} + a - \omega_g \frac{\cos(\omega_g T_e)}{\sin(\omega_g T_e)}\} + \frac{1}{\beta T_a}\{\frac{1}{\beta} + e^{-\beta(T_0-T_e)}(T_e - \frac{1}{\beta} - T_0)\} = 0 \\ \frac{\omega_g}{(a^2+\omega_g^2)\sin(\omega_g T_e)}(e^{-aT_e} + e^{-a(T_e-\frac{\pi}{\omega_g})}) + \frac{U_0}{E} = 0 \end{cases} \tag{20}$$

This is achieved by using a 2D Newton's method. The term e^{-aT_e} has been set into the () in order to avoid $e^{a\frac{\pi}{\omega_g}}$ overfloats during iteration. Initial value for a and E have empirically been set to $1.53419\,f0$ and $5.8251\,U_0 f_0$. The Newton's Gradient Descent applied on the non-linear system 20 converges most often in less than 10 iterations.

Eventually, we can compute:

$$U_g(t) = \begin{cases} F_a(\omega_g + a\sin(\omega_g t)e^{at} - \omega_g\cos(\omega_g t)e^{at}) & 0 < t < T_e \\ \frac{E}{\beta T_a}\{\frac{1}{\beta}e^{-\beta(t-T_e)} + e^{-\beta(T_0-T_e)}(t - \frac{1}{\beta} - T_0)\} & T_e < t < T_0 \end{cases} \tag{21}$$

2 Parameters

Equation 4 and system 20 do not have solutions for any values of parameters T_p ($\frac{\pi}{\omega_g}$), T_e and T_a, and they are commonly replaced by parameters more easy to handle. here we will use those proposed by Fant [1].

$$R_a = \frac{T_a}{T_0}; \quad R_k = \frac{T_e - T_p}{T_p}; \quad Rg = \frac{T_0}{2T_p} \tag{22}$$

The input parameter R_g can also be replaced by the opening quotient

$$OQ = \frac{1+R_k}{2R_g} = \frac{T_e}{T_0} \tag{23}$$

Or

$$OQ = \frac{1+R_k}{2R_g} + R_a \tag{24}$$

Fant also propose a single shape parameter R_d for controlling the glottis signal:

$$R_a = \frac{-1+4.8R_d}{100}; \quad R_k = \frac{22.4+11.8R_d}{100}; \quad R_g = \frac{0.25R_k(0.5+1.2R_k)}{0.11R_d - R_a(0.5+1.2R_k)} \tag{25}$$

References

[1] Gunnar Fant, *THE VOICE SOURCE MODELS AND PERFORMANCE*, Proceeding ICPhS 95 Stockholm, Vol 3: 82-89, 1995.